Applications of Fractional Calculus in PID Controller

Kaushal Khatri
*1(Asst. Professor, Department of Mathematics, Gh Raisoni college of engineering, Pune)
Kkhatri11@gmail.com*1

Abstract - In this paper, we discussed the history of fractional order calculus also the application of fractional calculus to control system. we demonstrated the better response of this Fractional order type of controllers, in comparison with the classical PID controllers. This paper deals with the tuning of conventional PID and fractional-order PID (FOPID) controllers by using Matlab and Simulink. The result shows the advantage of FOPID controller over Conventional PID controller. Podlubny proposed the concept of the fractional-order PID controllers in 1997 [1].

Keywords: Fractional order calculus, FOPID controller, PID Controller, Matlab Simulink.

1. INTRODUCTION

Fractional calculus is a more than 300 years old research topic, which deals with mathematics. The history of fractional calculus begins at the end of the 17th century. There is a number of Science and engineering applications where fractional calculus has been used. Fractional-order systems have lately been attracting significant attention and gaining more acceptance as generalizations to classical integer-order systems. Today, it is known that many real dynamic systems cannot be described by a system of simple differential equations of integer-order. In practice, such systems are encountered in electronics, signal processing, thermodynamics, biology, medicine, control theory, etc. This Action will favour scientific advancement in the above-mentioned areas by coordinating activities of academic research groups towards an efficient deployment of fractional theory to industry applications. The cooperation of researchers from different institutions will guarantee wide visibility of results from the Action. These mathematical phenomena allow us to describe a real object more accurately than the classical integer-order methods. Fractional calculus can be used in wide areas of engineering applications such as control theory in new fractional controllers and system models. The mathematics dealing with the derivatives and integrals of arbitrary non-integer order is more commonly known as fractional calculus. Various eminent mathematicians like L'hospital, Euler, Riemann and engineers and inventors like Heaviside, Caputo have made profound contributions to this interesting branch of mathematics. [1]

Fractional calculus is an extended form of integration & differentiation to non-integer order eg. Operator $D^\alpha$ here a and t are limit & $\alpha$ is the order. The order in calculus can also be a complex number called complex order calculus. Various Definitions are given by various mathematicians are as given below. Fractional calculus deals with the study of fractional order integrals and derivatives and their diverse applications Riemann–Liouville and Caputo are kinds of fractional derivatives [3].
Riemann Liouville fractional derivatives

\[
\mathcal{D}_t^\alpha f(t) = \begin{cases} 
\int_0^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} f(\tau) \, d\tau, & \text{if } \alpha \in \mathbb{R}^- \\
 f(t), & \text{if } \alpha = 0 \\
\frac{d^{[\alpha]} t}{dt^{[\alpha]}} D_t^{\alpha-\lfloor \alpha \rfloor} f(t), & \text{if } \alpha \in \mathbb{R}^+
\end{cases}
\]

Caputo fractional derivatives

\[
\mathcal{D}_t^\alpha f(t) = \begin{cases} 
\int_0^t \frac{(t-\tau)^{\alpha-1}}{\Gamma(\alpha)} f(\tau) \, d\tau, & \text{if } \alpha \in \mathbb{R}^- \\
 f(t), & \text{if } \alpha = 0 \\
\frac{d^{[\alpha]} t}{dt^{[\alpha]}} D_t^{\alpha-\lfloor \alpha \rfloor} f(t), & \text{if } \alpha \in \mathbb{R}^+
\end{cases}
\]

Grunwald–Letnikoff fractional derivatives

\[
\mathcal{D}_t^\alpha f(t) = \lim_{h \to 0^+} \frac{1}{h^\alpha} \sum_{k=1}^{[\frac{t}{h}]} (-1)^k \binom{\alpha}{k} f(t-kh)
\]

Some basic identities

a) It can be shown that for \( \alpha \neq \beta \) and \( f(k(a)) = 0 \) for \( k = 1,2,\ldots, \max\{n-1, m-1\} \)

\[
D^\beta_a D^\alpha_a f(x) = D^\alpha_a D^\beta_a f(x) = D^{\alpha+\beta}_a f(x)
\]

The commutation is also valid for the ordinary, integer order derivatives

\[
\frac{d^n}{dx^n} D^\alpha_a F(x) = D^\alpha_a \frac{d^n}{dx^n} F(x) = D^\alpha_a D^\beta_a f(x)
\]

The composition with fractional integrals looks like the following

\[
D^\alpha_a D^\beta_a f(x) = f(x)
\]

but of course, the opposite identity does not hold exactly [2]

\[
I^\beta_a D^\alpha_a f(x) = f(x) - \sum_{k=0}^{n} \frac{D^\alpha_a f(a)}{\Gamma(1+\beta-k)} (x-a)^{\beta-k}
\]

where \( n \leq 1 < \alpha \leq n \).

We do not have the same formulas for the derivative of a product or composite functions.

**Fractional Order Control Versus Integer Order Control**

The only way to know if it is better is to compare the optimal FO controller to its optimal IO controller counterpart. FOPID could satisfy at most 5 robustness criteria as compared to the usual classical PID controller which only have 3 parameters to be tuned for 3 robustness criteria. If you plan to satisfy 4 or 5 robustness criteria, then you should use FOPID because you can tune 5 parameters: \( K_p, K_i, K_d, \mu, \) and \( \delta \). Classical PID only has \( K_p, K_i, \) and \( K_d \). FOPID is more flexible at the point of control design but it is more complex to tune.

In this paper, we are going to discuss the application of fractional order in PID controller (the \( PI^\lambda D^\mu \)-controller). The transfer function of the plant is \( G_c(s) \) of this controller is defined as the ratio of the controller output \( U(s) \) and error \( E(s) \) so that we can write

\[
G_c(s) = \frac{U(s)}{E(s)} = K_p + K_i s^{-\lambda} + K_d s^{\lambda} \quad \lambda, \mu > 0
\]

The corresponding output \( u(t) = L^{-1}\{U(s)\} \) for the \( PI^\lambda D^\mu \)-controller in the time domain is given...
by $u(t) = K_pe(t) + K_iD^{-\lambda}e(t) + K_DE^\mu e(t)$

When $\lambda = \mu = 1$, the above results reduce to those of the classical PID controller.

When $\lambda = 1$ and $\mu = 0$, we obtain the corresponding results for PI-controller, while $\lambda = 0$ and $\mu = 1$ leads to the PD-controller. All PID-controllers are special cases of the fractional $PI^\lambda D^\mu$ controller described by its transfer function. Numerous applications have demonstrated that $PI^\lambda D^\mu$ controllers perform sufficiently better for the control of fractional-order dynamical systems than the classical PID-controllers.[3]

2. The Applicative Point of View

Fractional calculus has a long history, but, from an applicative point of view, it fell into oblivion for many years since it was considered not useful for solving problems in physics and engineering. Actually, this oblivion was due to its high complexity and the lack of an acceptable physical and geometric interpretation. Just in 2002, Podlubny proposed a convincing explanation of the fractional phenomena [4]. He suggested both a geometric interpretation of the Riemann-Liouville fractional integral (based on the projection of a very fascinating “shadow on the wall”) and a physical interpretation for the Riemann-Liouville (and Caputo) fractional differentiation (based on the fact that, as in the theory of relativity, two time scales should be considered simultaneously: the ideal, equably flowing homogeneous time and the cosmic inhomogeneous time).[5]

Survey of applications

There are many fields of applications where we can use the fractional calculus, as for examples:

- Visco elasticity
- Control theory
- Heat conduction
- Electricity
- Mechanics
- Chaos and Fractals, etc.

Application to a fractional order system:

Let the Single input single output system, The transfer function of the plant given by,

$$G(s) = \frac{3 + s^{0.25}}{1 + s^{0.25} + s^{0.5} + s^{0.75}}$$

![Fig 2: Unit Step responses of the system with PID and FOPID controllers.](image)

By using Matlab we obtain optimized parameters for PID and FOPID. In Matlab, there is a PID tuning tool available for PID parameters and for FOPID we used FOMCON tool.

Conclusion

The Fraction order Control PID can be used for both fractional-order and Integer-order control. The designing FOPID controller is
difficult compared to the PID controller since the FOPID controller is including more parameters. The simulation results show that the FOPID is preferable and more robust than a traditional PID in terms of the degree of stability. From Fig 2, we conclude that control efforts of FOPID are very lesser than the classical PID controller.

REFERENCES